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A STUDY ON THE POSITIVE PELL EQUATION

$y^2 = 42x^2 + 7$

J.Shanthi[1], P.Deepalakshmi[2], M.A. Gopalan[3]

[1]*Assistant Professor, Department of Mathematics, Shrimathi indira Gandhi college, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.*

[2]*PG Scholar, Department of Mathematics, Shrimathi indira Gandhi college , Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.*

[3]*Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.*

Abstract: *This paper concerns with the problem of obtaining non-zero distinct integer solutions to the positive pell equation represented by the binary quadratic equation* $y^2 = 42x^2 + 7$ *. A few interesting relations among the solutions are presented. Further, by considering suitable linear combinations among the solutions of the considered hyperbola, the other choices of hyperbolas, parabolas, pythagorian triangle,*2^{nd}*order Ramanujan numbers, sequence of diophantine 3-tuples with suitable property are presented.*

Keywords: **Positive pell equation, binary quadratic, hyperbola, parabola, pythagorian triangle,2^{nd} order Ramanujan numbers, sequence of diophantine 3-tuples.**

1. INTRODUCTION

One of the areas of Number theory that has attracted many mathematicians since antiquity is the subject of diophantine equations. A diophantine equation is a polynomial equation in two or more unknowns such that only the integer solutions are determined. No doubt that diophantine equation possess supreme beauty and it is the most powerful creation of the human spirit. A pell equation is a type of non-linear diophantine equation in the form

$y^2 - Dx^2 = \pm 1$ where $D > 0$ and square-free. The above equation is also called the Pell-Fermat equation. In Cartesian co-ordinates, this equation has the form of a hyperbola. The binary quadratic diophantine equation having the form

$$y^2 = Dx^2 + N \ (N > 0\ ;\ D > 0, \text{ a non-square integer})$$

is referred to as the positive form of the Pell equation and one may refer [1, 9] for a few illustrations on Pell equation.

As quadratic Diophantine equations are rich in variety,the above references motivated us for determining integer solutions to other choices of positive Pell equation.

In this communication ,the positive Pell equation $\boldsymbol{y^2 = 42x^2 + 7}$ is considered for obtaining non-zero distint integer solutions. Further, by considering suitable linear combinations among the solutions of the considered hyperbola, the other choices of hyperbolas, parabolas, pythagorian triangle,$\boldsymbol{2^{nd}}$order Ramanujan numbers, sequence of diophantine 3-tuples with suitable property are presented.

Method of analysis:

The hyperbola represented by the non-homogeneous quadratic equation under consideration is

$$y^2 = 42x^2 + 7 \tag{1}$$

The smallest positive integer solution is x_0=1, y_0=7

To find the other solutions to (1),consider the corresponding pellian equation is given by

$$y^2 = 42x^2 + 1 \tag{2}$$

where the general solution $(\widetilde{x_n}, \widetilde{y_n})$ is

$$\widetilde{y_n} = \frac{1}{2} f_n$$

$$\widetilde{x_n} = \frac{1}{2\sqrt{D}} g_n$$

Where

$$f_n = \left(13 + 2\sqrt{42}\right)^{n+1} + \left(13 - 2\sqrt{42}\right)^{n+1}$$

$$g_n = \left(13 + 2\sqrt{42}\right)^{n+1} - \left(13 - 2\sqrt{42}\right)^{n+1}$$

- Employing the lemma of Brahmagupta between the solutions (x_0, y_0) &$(\widetilde{x_n}, \widetilde{y_n})$,the general solution (x_{n+1}, y_{n+1}) to (1) is given by

$$x_{n+1} = x_0\widetilde{y_n} + y_0\widetilde{x_n}$$

$$= 1 * \frac{1}{2}f_n + 7 * \frac{1}{2\sqrt{42}}g_n$$

$$y_{n+1} = y_0\widetilde{y_n} + Dx_0\widetilde{x_n}$$

$$= 7 * \frac{1}{2}f_n + 42 * 1 * \frac{1}{2\sqrt{42}}g_n$$

where n=-1,0,1,……

A few numerical solutions to **(1)** are presented in table below:

Table: Numerical solutions

n	x_{n+1}	y_{n+1}
-1	1	7
0	27	175
1	701	4543
2	18199	117943
3	472473	3061975
4	12266099	79493407
5	318446101	2063766607

Observations:

- The values of x_n and y_n are odd
- $x_{3n-2} \equiv 0(mod3),\ y_{n-1} \equiv 0(mod7)$ where n=1,2,3,…….
- A few interesting relations among the solutions are given below:

  - $x_{n+3} - 26x_{n+2} + x_{n+1}$
  - $y_{n+3} - 26y_{n+2} + y_{n+1}$
  - $2y_{n+1} = x_{n+2} - 13x_{n+1}$
  - $2y_{n+2} = 13x_{n+2} - x_{n+1}$
  - $2y_{n+2} = x_{n+3} - 13x_{n+2}$
  - $2y_{n+3} = 337x_{n+2} - 13x_{n+1}$
  - $2y_{n+1} = 13x_{n+3} - 337x_{n+2}$
  - $2y_{n+3} = 13x_{n+3} - x_{n+2}$
  - $168x_{n+2} = y_{n+3} - y_{n+1}$

- $84x_{n+2} = 13y_{n+2} - y_{n+1}$
- $84x_{n+2} = y_{n+3} - 13y_{n+2}$
- $84x_{n+1} = y_{n+2} - 13y_{n+1}$
- $84x_{n+3} = 337y_{n+2} - 13y_{n+1}$
- $84x_{n+1} = 13y_{n+3} - 337y_{n+2}$
- $84x_{n+3} = 13y_{n+3} - y_{n+2}$
- $4y_{n+2} = x_{n+3} - x_{n+1}$
- $52y_{n+3} = 337x_{n+3} - x_{n+1}$
- $52y_{n+1} = x_{n+3} - 337x_{n+1}$
- $2184x_{n+1} = y_{n+3} - 337y_{n+1}$
- $2184x_{n+3} = 337y_{n+3} - y_{n+1}$

➢ Expressions representing square integers:

- $\frac{1}{7}[175x_{2n+4} - 4543x_{4n+3} + 14]$
- $x_{2n+3} - 25x_{2n+2} + 2$
- $\frac{1}{7}[27y_{2n+2} - y_{2n+3} + 14]$
- $\frac{1}{7}[701y_{2n+3} - 27y_{2n+4} + 14]$
- $\frac{2}{91}[175y_{2n+2} - 42x_{2n+3} + 91]$
- $\frac{2}{7}[175y_{2n+3} - 1134x_{2n+3} + 7]$
- $\frac{2}{91}[175y_{2n+4} - 29442x_{2n+3} + 91]$
- $\frac{2}{91}[4543y_{2n+3} - 1134x_{2n+4} + 91]$
- $\frac{2}{7}[4543y_{2n+4} - 29442x_{2n+4} + 7]$
- $\frac{1}{182}[701y_{2n+2} - y_{2n+4} + 364]$
- $\frac{1}{26}[x_{2n+4} - 649x_{2n+2} + 52]$
- $2[y_{2n+2} - 6x_{2n+2} + 1]$
- $\frac{2}{13}[y_{2n+3} - 162x_{2n+2} + 13]$
- $\frac{2}{337}[y_{2n+4} - 4206x_{2n+2} + 337]$
- $\frac{2}{2359}[4543y_{2n+2} - 42x_{2n+4} + 2359]81$

➢ Expressions representing cubical integers:

- $x_{3n+4} - 25x_{3n+2} + 3x_{n+2} - 75x_{n+1}$
- $\frac{1}{7}[175x_{3n+5} - 4543x_{3n+4} + 525x_{n+3} - 13629x_{n+2}]$

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- $\frac{1}{7}[27y_{3n+3}-y_{3n+4}+81y_{n+1}-3y_{n+2}]$
- $\frac{1}{7}[701y_{3n+4}-27y_{3n+5}+2103y_{n+2}-81y_{n+3}]$
- $\frac{2}{91}[175y_{3n+3}-42x_{3n+4}+525y_{n+1}-126x_{n+2}]$
- $\frac{2}{7}[175y_{3n+4}-1134x_{3n+4}+525y_{n+2}-3402x_{n+2}]$
- $\frac{2}{91}[175y_{3n+5}-29442x_{3n+4}+525y_{n+3}-88326x_{n+2}]$
- $\frac{2}{91}[4543y_{3n+4}-1134x_{3n+5}+13129y_{n+2}-3402x_{n+3}]$
- $\frac{2}{7}[4543y_{3n+5}-29442x_{3n+5}+13629y_{n+3}-88326x_{n+3}]$
- $\frac{1}{182}[701y_{3n+3}-y_{3n+5}+2103y_{n+1}-3y_{n+3}]$
- $\frac{1}{26}[x_{3n+5}-649x_{3n+3}+3x_{n+3}-1947x_{n+1}]$
- $2[y_{3n+3}-6x_{3n+3}+3y_{n+1}-18x_{n+1}]$
- $\frac{2}{13}[y_{3n+4}-162x_{3n+3}+3y_{n+2}-486x_{n+1}]$
- $\frac{2}{337}[y_{3n+5}-4206x_{3n+3}+3y_{n+3}-12618x_{n+1}]$
- $\frac{2}{2359}[4543y_{3n+3}-42x_{3n+5}+13629y_{n+1}-126x_{n+3}]$

➢ Expressions representing biquadratic integers:

- $x_{4n+5}-25x_{4n+4}+4f_n^2-2$
- $x_{4n+5}-25x_{4n+4}+4x_{2n+3}-100x_{2n+2}+6$
- $\frac{1}{7}[175x_{4n+6}-4543x_{4n+5}]+4f_n^2-2$
- $\frac{1}{7}[175x_{4n+6}-4543x_{4n+5}+700x_{2n+4}-18172x_{4n+3}+42]$
- $\frac{1}{7}[27y_{4n+4}-y_{4n+5}]+4f_n^2-2$
- $\frac{1}{7}[27y_{4n+4}-y_{4n+5}+108y_{2n+2}-4y_{2n+3}+42]$
- $\frac{1}{7}[701y_{4n+5}-27y_{4n+6}]+4f_n^2-2$
- $\frac{1}{7}[701y_{4n+5}-27y_{4n+6}+2804y_{2n+3}-108y_{2n+4}+42]$
- $\frac{2}{91}[175y_{4n+4}-42x_{4n+5}]+4f_n^2-2$
- $\frac{2}{91}[175y_{4n+4}-42x_{4n+5}+700y_{2n+2}-168x_{2n+3}+273]$
- $\frac{2}{7}[175y_{4n+5}-1134x_{4n+5}]+4f_n^2-2$
- $\frac{2}{7}[175y_{4n+5}-1134x_{4n+5}+700y_{2n+3}-4536x_{2n+3}+21]$
- $\frac{2}{91}[175y_{4n+6}-29442x_{4n+5}]+4f_n^2-2$
- $\frac{2}{91}[175y_{4n+6}-29442x_{4n+5}+700y_{2n+4}-117768x_{2n+3}+273]$
- $\frac{2}{91}[4543y_{4n+5}-1134x_{4n+6}]+4f_n^2-2$
- $\frac{2}{91}[4543y_{4n+5}-1134x_{4n+6}+18172y_{2n+3}-4536x_{2n+4}+273]$

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- $\frac{2}{7}[4543y_{4n+6} - 29442x_{4n+6}] + 4f_n^2 - 2$
- $\frac{2}{7}[4543y_{4n+6} - 29442x_{4n+6} + 18712y_{2n+4} - 117768x_{2n+4} + 21]$
- $\frac{1}{182}[701y_{4n+4} - y_{4n+6}] + 4f_n^2 - 2$
- $\frac{1}{182}[701y_{4n+4} - y_{4n+6} + 2804y_{2n+2} - 4y_{2n+4} + 1902]$
- $2[y_{4n+4} - 6x_{4n+4} + 4y_{2n+2} - 24x_{2n+2} + 3]$
- $\frac{1}{26}[x_{4n+6} - 649x_{4n+4}] + 4f_n^2 - 2$
- $\frac{1}{26}[x_{4n+6} - 649x_{4n+4} + 4x_{2n+4} - 2596x_{2n+2} + 156]$
- $\frac{2}{13}[y_{4n+5} - 162x_{4n+4}] + 4f_n^2 - 2$
- $\frac{2}{13}[y_{4n+5} - 162x_{4n+4} + 4y_{2n+3} - 648x_{2n+2} + 39]$
- $\frac{2}{337}[y_{4n+6} - 4206x_{4n+4}] + 4f_n^2 - 2$
- $\frac{2}{337}[y_{4n+6} - 4206x_{4n+4} + 4y_{2n+4} - 16824x_{2n+2} + 1011]$
- $\frac{2}{2359}[4543y_{4n+4} - 42x_{4n+6}] + 4f_n^2 - 2$
- $\frac{2}{2359}[4543y_{4n+4} - 42x_{4n+6} + 18172y_{2n+2} - 168x_{2n+4} + 7077]$

➢ Employing linear combinations among the solutions, one obtains solutions to other choices of hyperbolas

Choice1: Let $X = x_{n+2} - 25x_{n+1}, Y = 27x_{n+1} - x_{n+2}$
$$49X^2 = 2[98 + 21Y^2]$$
Note that (X, Y) satisfies the hyperbola

Choice2: Let $X = 175x_{n+3} - 4543x_{n+2}, Y = 701x_{n+2} - 27x_{n+3}$
$$X^2 = 2[98 + 21Y^2]$$
Note that (X, Y) satisfies the hyperbola

Choice3: Let $X = 27y_{n+1} - y_{n+2}, Y = y_{n+2} - 25y_{n+1}$
$$6X^2 - 7Y^2 = 1176$$
Note that (X, Y) satisfies the hyperbola

Choice4: Let $X = 701y_{n+2} - 27y_{n+3}, Y = 25y_{n+3} - 649y_{n+2}$
$$6X^2 = 7[168 + Y^2]$$
Note that (X, Y) satisfies the hyperbola

Choice5: Let $X = 175y_{n+1} - 42x_{n+2}, Y = 7x_{n+2} - 27y_{n+1}$
$$X^2 = 7[1183 + 6Y^2]$$
Note that (X, Y) satisfies the hyperbola

Choice6: Let $X = 175y_{n+2} - 1134x_{n+2}, Y = 175x_{n+2} - 27y_{n+2}$
$$X^2 = 7[7 + 6Y^2]$$
Note that (X, Y) satisfies the hyperbola

Choice7: Let $X = 175y_{n+3} - 29442x_{n+2}, Y = 4543x_{n+2} - 27y_{n+3}$
$$X^2 = 7[1183 + 6Y^2]$$

Note that (X, Y) satisfies the hyperbola

Choice8: Let $X = 4543y_{n+2} - 1134x_{n+3}, Y = 175x_{n+3} - 701y_{n+2}$

$$X^2 = 7[1183 + 6Y^2]$$

Note that (X, Y) satisfies the hyperbola

Choice9: Let $X = 4543y_{n+3} - 29442x_{n+3}, Y = 4543x_{n+3} - 701y_{n+1}$

$$X^2 = 7[7 + 6Y^2]$$

Note that (X, Y) satisfies the hyperbola

Choice10: Let $X = 701y_{n+1} - y_{n+3}, Y = y_{n+3} - 649y_{n+1}$

$$6[X^2 - 132496] = 7Y^2$$

Note that (X, Y) satisfies the hyperbola

Choice11: Let $X = x_{n+3} - 649x_{n+1}, Y = 701x_{n+1} - x_{n+3}$

$$6Y^2 = 7[X^2 - 2704]$$

Note that (X, Y) satisfies the hyperbola

Choice12: Let $X = y_{n+1} - 6x_{n+1}, Y = 7x_{n+1} - y_{n+1}$

$$7[X^2 - 1] = 6Y^2$$

Note that (X, Y) satisfies the hyperbola

Choice13: Let $X = y_{n+2} - 162x_{n+1}, Y = 175x_{n+1} - y_{n+2}$

$$7[X^2 - 169] = 6Y^2$$

Note that (X, Y) satisfies the hyperbola

Choice14: Let $X = y_{n+3} - 4206x_{n+1}, Y = 4543x_{n+1} - y_{n+3}$

$$7[X^2 - 113569] = 6$$

Note that (X, Y) satisfies the hyperbola

Choice15: Let $X = 4543y_{n+1} - 42x_{n+3}, Y = 7x_{n+3} - 701y_{n+1}$

$$X^2 = 7[6Y^2 + 794983]$$

Note that (X, Y) satisfies the hyperbola

- Employing linear combinations among the solutions, one obtains solutions to other choices of parabolas

Choice1: Let $X = x_{2n+3} - 25x_{2n+2} + 2, Y = 27x_{n+1} - x_{n+2}$

$$49X = 2[392 - 21Y^2]$$

Note that (X, Y) satisfies the parabola

Choice2: Let $X = 175x_{2n+4} - 4543x_{4n+3} + 14, Y = 701x_{n+2} - 27x_{n+3}$

$$X - 6Y^2 = 28$$

Note that (X, Y) satisfies the parabola

Choice3: Let $X = 27y_{2n+2} - y_{2n+3} + 14, Y = y_{n+2} - 25y_{n+1}$

$$X = 2[3Y^2 + 14]$$

Note that (X, Y) satisfies the parabola

Choice4: Let $X = 701y_{2n+3} - 27y_{2n+4} + 14, Y = 25y_{n+3} - 649y_{n+2}$

$$6[X - 28] = Y^2$$

Note that (X, Y) satisfies the parabola

Choice5: Let $X = 175y_{2n+2} - 42x_{2n+3} + 91, Y = 7x_{n+2} - 27y_{n+1}$

$$13X = 2[1183 + 6Y^2]$$

Note that (X, Y) satisfies the parabola

Choice6: Let $X = 175y_{2n+3} - 1134x_{2n+3}, Y = 175x_{n+2} - 27y_{n+2}$

$X = 2[7 + 6Y^2]$

Note that (X, Y) satisfies the parabola

Choice7: Let $X = 175y_{2n+4} - 29442x_{2n+3} + 91, Y = 175x_{n+2} - 27y_{n+2}$

$13[X - 182] = 12Y^2$

Note that (X, Y) satisfies the parabola

Choice8: Let $X = 4543y_{2n+3} - 1134x_{2n+4} + 91, Y = 175x_{n+3} - 701y_{n+2}$

$13[X - 182] = 12Y^2$

Note that (X, Y) satisfies the parabola

Choice9: Let $X = 4543y_{2n+4} - 29442x_{2n+4} + 7, Y = 4543x_{n+3} - 701y_{n+3}$

$X = 2[6Y^2 + 7]$

Note that (X, Y) satisfies the parabola

Choice10: Let $X = 701y_{2n+2} - y_{2n+4} + 364, Y = y_{n+3} - 649y_{n+1}$

$13[X - 48] = 6Y^2$

Note that (X, Y) satisfies the parabola

Choice11: Let $X = x_{2n+4} - 649 + 52, Y = 701x_{n+1} - x_{n+3}$

$91[X - 104] = 3Y^2$

Note that (X, Y) satisfies the parabola

Choice12: Let $X = y_{2n+2} - 6x_{2n+2} + 1, Y = 7x_{n+1} - y_{n+1}$

$7[X - 2] = 12Y^2$

Note that (X, Y) satisfies the parabola

Choice13: Let $X = y_{2n+3} - 162x_{2n+2} + 13, Y = 175x_{n+1} - y_{n+2}$

$91[X - 26] = 12Y^2$

Note that (X, Y) satisfies the parabola

Choice14: Let $X = y_{2n+4} - 4206x_{2n+2} + 337, Y = 4543x_{n+1} - y_{n+3}$

$337[X - 674] = 2Y^2$

Note that (X, Y) satisfies the parabola

Choice15: Let $X = 4543y_{2n+2} - 42x_{2n+4} + 2359, Y = 7x_{n+3} - 701y_{n+1}$

$12Y^2 = 337[X - 4718]$

Note that (X, Y) satisfies the parabola

Generation of pythogoriean triangle:

Let p,q be any two non-zero distinct integers.

Assume p=$x_{n+1} + y_{n+1}$

q=x_{n+1}

Note that p>q>0.

Treat p,q as the generation of pythogorian triangle (X,Y,Z)

where X=2pq,Y=$p^2 - q^2$,Z=$p^2 + q^2$,p>q>0 at A,P denote its area and perimeter.

Then the following relations are observed:

- $X - 21Y + 20Z + 7$

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- $\frac{2A}{P} = x_{n+1}y_{n+1}$
- $\frac{4A}{P} = X + Y - Z$

➢ Considering suitable values of $x_n \& y_n$, one generates 2^{nd} order Ramanujan numbers with base integers as real integers

For illustration, consider

$y_1 = 175 = 1 \times 175 = 35 \times 5 = 25 \times 7$ (*)

Now, $1 \times 175 = 35 \times 5$

$\rightarrow (175+1)^2 + (35-5)^2 = (175-1)^2 + (35+5)^2$

$\rightarrow 176^2 + 30^2 = 174^2 + 40^2 = 31876$

$1 \times 175 = 25 \times 7$

$\rightarrow (175+1)^2 + (25-7)^2 = (175-1)^2 + (25+7)^2 = 31300$

$35 \times 5 = 25 \times 7$

$\rightarrow (35+5)^2 + (25-7)^2 = (25+7)^2 + (35-5)^2 = 1924$

Note:

$1 \times 175 = 35 \times 5$

$\rightarrow 88^2 - 87^2 = 20^2 - 15^2$

$\rightarrow 88^2 + 15^2 = 20^2 + 87^2 = 7969$

$1 \times 175 = 25 \times 5$

$\rightarrow 88^2 - 87^2 = 16^2 - 9^2$

$\rightarrow 88^2 + 9^2 = 16^2 + 87^2 = 7825$

$35 \times 5 = 25 \times 7$

$\rightarrow 20^2 - 15^2 = 16^2 - 9^2$

$\rightarrow 20^2 + 9^2 = 16^2 + 15^2 = 170$

Thus 31876,31300,1924,7969,7825,170 represent 2^{nd} order Ramanujan numbers

Now consider,

$x_1 = 27 = 27 \times 1 = 3 \times 9$ (*)

$\rightarrow (27+1)^2 + (9-3)^2 = (9+3)^2 + (27-1)^2 = 820$

Note:

$1 \times 27 = 3 \times 9$

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$\rightarrow 14^2 - 13^2 = 6^2 - 3^2$
$\rightarrow 14^2 + 3^2 = 6^2 + 13^2 = 205$

Thus 820,205 represent 2^{nd} order Ramanujan numbers

- Considering suitable values of $x_n \& y_n$, one generates 2^{nd} order Ramanujan numbers with base integers as guassian integers

For illustration, consider again x_1, y_1 represented by (*),

Now, $1 \times 27 = 3 \times 9 \rightarrow (1 + i27)^2 + (9 - i3)^2 = -656$
Also, $1 \times 27 = 3 \times 9 \rightarrow (27 + i)^2 + (3 - i9)^2 = 656$

Now, $1 \times 175 = 35 \times 5 \rightarrow (1 + i175)^2 + (5 - i35)^2 = -31824$
Also, $1 \times 175 = 35 \times 5 \rightarrow (175 + i1)^2 + (35 - i5)^2 = 31824$

Note that -656,656,-31824 & 31824 represent 2^{nd} order Ramanujan numbers with base integers as gaussian integers.
In a similar manner, other 2^{nd} order Ramanujan numbers are obtained

Formation of sequence of Diophantine 3-tuples:

Consider the solution to (1) given by

$$x_0 = 1, y_0 = 7$$

It is observed that

$$x_0 y_0 + k^2 - 7 = k^2, \text{ a perfect square}$$

The pair (x_0, y_0) represents diaphantine 2-tuple with property D$(k^2 - 7)$.

If c is the 3^{rd} tuple, then it satisfies the system of double equations.

$$c + k^2 - 7 = p^2$$
$$7c + k^2 - 7 = q^2$$

Eliminating c between (1) and (2) ,we have

$$6(k^2 - 7) = 7p^2 - q^2 \tag{3}$$

Taking,

$$p = X + T, q = X + 7T \tag{4}$$

In(3) and simplying, we get

$$X^2 = k^2 - 7 + 7T^2$$

which is satisfied by,

$$T = 1, X = k$$

In view of (4) and (1), it is seen that

$$c = 2k + 8$$

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Note that (1,7,2k+8) represents diaphantine 3-tuple with property D(k^2-7)

The process of obtaining sequences of diaphantine 3-tuples with property D(k^2-7) is illustrated below:

Let M be a 3*3 square matrix given by

$$M=\begin{pmatrix}1 & 0 & 2\\ 0 & 0 & -1\\ 0 & 1 & 2\end{pmatrix}$$

Now, (1,7,2k+8)M=(1,2k+8,4k+11)

Note that

1*(2k+8)+k^2-7=perfect square
1*(4k+11)+k^2-7=perfect square
(2k+8)*(4k+11)+k^2-7=perfect square

Therefore the triple (1,2k+8,4k+11) represents diaphantine 3-tuple with property D(k^2-7).The repetition of the above process leads to sequences of diaphantine 3-tuples whose general form (1,c_{s-1},c_s) is given by

(1,$s^2+(2k-2)-2k+8, s^2+2ks+7$),s=1,2,3……..

A few numerical illustrations are given in Table below:

Table:Numerical illustrations

k	(1,c_0,c_1)	(1,c_1,c_2)	(1,c_2,c_3)	D(k^2-7)
0	(1,7,8)	(1,8,11)	(1,11,16)	D(-7)
1	(1,7,10)	(1,10,15)	(1,15,22)	D(-6)
2	(1,7,12)	(1,12,19)	(1,19,28)	D(-3)

It is noted that the triple (c_{s-1},c_s+1,c_{s+1}) ,s=1,2,3……
forms an arithmetic progression.

In a similar way one may generate sequences of diaphantine 3-tuples with suitable property through the other solutions to (1).

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