International Research Journal of Education and Technology
ISSN 2581-7795

ON MICRO GRILL FORMS OF OPEN SETS

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Abstract. In this paper, we introduce new concepts called Φ_m-open, $\mathcal{G}$-m-α-open, $\mathcal{G}$-m-pre-open, $\mathcal{G}$-m-semi-open, $\mathcal{G}$-m-b-open, $\mathcal{G}$-m-β-open, $\mathcal{G}$-m-regular-open, $\mathcal{G}$-m-π-open, which are simple forms of micro open sets in an micro grill topological spaces. Also we characterize the relations between them and the related properties.

Keywords : $n\mathcal{I}_{\ddot{g}}$-closed sets, $n\mathcal{I}_{\ddot{g}}$-lc-sets , $n\mathcal{I}_{\ddot{g}}$-lc*-sets , $n\mathcal{I}_{\ddot{g}}$-lc**-sets and $n\mathcal{I}_{\ddot{g}}$-LC-continuous.

1. Introduction

The idea of grill on a topological space was first introduced by Choquet [11] in 1947. It is observed from literature that the concept of grills is a powerful supporting tool, like nets and filters, in dealing with many a topological concept quite effectively. A number of theories and features has been handled in [1, 29, 33]. It helps to expand the topological structure which is used to measure the description rather than quantity, such as love, intelligence, beauty, quality of education and etc. In [34], Roy and Mukherjee defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space. The notion of a micro

[0]Volume: 2, Issue: 1, (2021), Pages: 10 - 26

topology was introduced and studied by Chandrasekar [9] which was defined Micro closed, Micro open, Micro interior and Micro closure. S. Ganesan [13] introduced and studied Micro regular open in micro topological spaces. S. Ganesan [17] introduced and studied new type of micro grill topological spaces via micro grills and $m_{\mathcal{G}}$g-closed sets. In this paper is to introduce new concepts called Φ_m-open, $\mathcal{G}$-m-α-open, $\mathcal{G}$-m-pre-open, $\mathcal{G}$-m-semi-open, $\mathcal{G}$-m-b-open, $\mathcal{G}$-m-β-open, $\mathcal{G}$-m-regular-open, $\mathcal{G}$-m-π-open, which are simple forms of micro open sets in an micro grill topological spaces. Also we characterize the relations between them and the related properties.

2. Preliminary

Definition 2.1. [32] *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. $Thatis, L_R(X) = \bigcup_{x\in U}$ {R(X): R(X) $\subseteq$ X} where R(x) denotes the equivalence class determined by X.*

(2) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. $Thatis, U_R(X) = \bigcup_{x\in U}$ {R(X): R(X) $\cap$ X $\neq \phi$}.*

(3) *The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$. $Thatis, B_R(X) = U_R(X) - L_R(X)$.*

Property 2.2. [32] *If (U, R) is an approximation space and X, $Y \subseteq U$, then*

(1) $L_R(X) \subseteq X \subseteq U_R(X)$.

(2) $L_R(\phi) = U_R(\phi) = \phi$, $L_R(U) = U_R(U) = U$.

(3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.

(4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.

(5) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.

(6) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.

(7) $L_R(X) \subseteq L_R(Y) and U_R(X) \subseteq U_R(Y) whenever X \subseteq Y$.

(8) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.

(9) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$.

(10) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$.

Definition 2.3. [32] *Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\} where X \subseteq U$. Then by Property 2.2, $\tau_R(X)$ satisfies the following axioms*

(1) *U , $\phi \epsilon \tau_R(X)$.*

(2) *The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.*

(3) *The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.*

Then $\tau_R(X)$ is called the Nano topology on U with respect to X.

The space $(U, \tau_R(X))$ is the Nano topological space. The elements of are called Nano open sets.

Definition 2.4. [32]

If $(U, \tau_R(X))$ is the Nano topological space with respect to X where $X \subseteq U$ and if $M \subseteq U$, then

(1) *The Nano interior of the set M is defined as the union of all Nano open subsets contained in M and it is denoted by NInte(M). That is, NInte(M) is the largest Nano open subset of M.*
(2) *The Nano closure of the set M is defined as the intersection of all Nano closed sets containing M and it is denoted by NClo(M). That is, NClo(M) is the smallest Nano closed set containing M.*

Definition 2.5. [11] *A collection $\mathcal{G}$ of a nonempty subsets of a topological space X is called a grill on X if (1) $A \in \mathcal{G}$ and $A \subseteq B \subseteq X \Rightarrow B \in \mathcal{G}$, (2) A, $B \subseteq X$ and A $\cup$ $B \in \mathcal{G} \Rightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$. For any point x of a topological space (X, τ), we shall let $\tau(x)$ denote the collection of all open neighbourhoods of x.*

Definition 2.6. [9] *Let (U, $\tau_R(X)$) be a nano topological space. Then, $\mu_R(X) = \{N \cup (\acute{N} \cap \mu) : N, \acute{N} \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.*

Definition 2.7. [9] *The Micro interior of a set A is denoted by Micro-int(A) (briefly, m-int(A)) and is defined as the union of all Micro open sets contained in A. i.e., m-int(A) = $\cup$ $\{G : G$ is Micro open and $G \subseteq A\}$.*

Definition 2.8. [9] *The Micro closure of a set A is denoted by Micro-cl(A) (briefly, m-cl(A)) and is defined as the intersection of all Micro closed sets containing A. i.e., Mic-cl(A) = $\cap$ $\{F : F$ is Micro closed and $A \subseteq F\}$.*

Definition 2.9. [9] *For any two Micro sets A and B in a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$),*

(1) *A is a Micro closed set if and only if Mic-cl(A) = A.*

(2) *A is a Micro open set if and only if Mic-int(A) = A.*

(3) $A \subseteq B$ *implies Mic-int(A)* $\subseteq$ *Mic-int(B) and Mic-cl(A)* $\subseteq$ *Mic-cl(B).*

(4) *Mic-cl(Mic-cl(A)) = Mic-cl(A) and Mic-int(Mic-int(A)) = Mic-int(A).*

(5) *Mic-cl* $(A \cup B) \supseteq$ *Mic-cl(A)* $\cup$ *Mic-cl(B).*

(6) *Mic-cl*$(A \cap B) \subseteq$ *Mic-cl(A)* $\cap$ *Mic-cl(B).*

(7) *Mic-int*$(A \cup B) \supseteq$ *Mic-int(A)* $\cup$ *Mic-int(B).*

(8) *Mic-int*$(A \cap B) \subseteq$ *Mic-int(A)* $\cap$ *Mic-int(B).*

(9) *Mic-cl*$(\mathcal{A}^C) = [Mic - int(A)]^C$.

(10) *Mic-int*$(\ A^C) = [Mic - cl(A)]^C$

Definition 2.10. *Let* $(U, \tau_R(X), \mu_R(X))$ *be a micro topological space and* $A \subseteq U$*. Then,*

(1) *A is called Micro* α*-open if* $A \subseteq$ *Mic-int(Mic-cl(Mic-int(A)))* [10].

(2) *A is called Micro pre-open if* $A \subseteq$ *Mic-int(Mic-cl(A))* [9].

(3) *A is called Micro semi-open if* $A \subseteq$ *Mic-cl(Mic-int(A))* [9].

(4) *A is called Micro b-open if* $A \subseteq$ *Mic-int(Mic-cl(A))* $\cup$ *Mic-cl(Mic-int(A))* [30].

(5) *A is called Micro* β*-open if* $A \subseteq$ *Mic-cl(Mic-int(Mic-cl(A))).* [30]

(6) *A is called Micro regular-open if A = Mic-int(Mic-cl(A))* [13, 15].

(7) *A is called Micro* π*-open set if A is the finite union of Micro regular-open sets* [13].

The complement of above mentioned micro open sets are called their respective micro closed sets.

Let (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space, where $\mathcal{N} = \tau_R(X)$ and $\mathcal{M} = \mu_R(X)$ and it is denoted by (K, $\mathcal{M}$).
Let (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space and $\mathcal{G}$ be a grill on K is called a micro grill topological space and it is denoted by (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$) or (K, $\mathcal{M}$, $\mathcal{G}$).
Let a space K we shall mean a micro grill topological spaces (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$). Also, the power set of K will be written as $\wp$(K), we shall let $\mathcal{M}(k)$ to stand for the collection of all micro open neighbourhoods of k.

Definition 2.11. [17] *Let (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space and $\mathcal{G}$ be a grill on K. We define a mapping $\Phi_m : \wp(K) \to \wp(K)$, denoted by $\Phi_{m\mathcal{G}}(A, \mathcal{M})$ (for $A \in \wp(K)$) or $\Phi_{m\mathcal{G}}(A)$ or simply by $\Phi_m(A)$(when it is known which micro topology and grill on K we are talking about), called the operator associated with the grill $\mathcal{G}$ and the micro topology $\mathcal{M}$, and is defined by $\Phi_m(A) = \Phi_{m\mathcal{G}}(A, \mathcal{M}) = \{k \in K : U \cap A \in \mathcal{G}, \forall\ U \in \mathcal{M}(k)\}$.*

Note 2.12. [17] *(K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space with a grill $\mathcal{G}$ on K and for every A , B be subsets of K. Then*

(1) $A \subseteq B\ (\subseteq K) \Rightarrow \Phi_m(A) \subseteq \Phi_m(B)$,
(2) $\mathcal{G}_1 \subseteq \mathcal{G}_2 \Rightarrow \Phi_{m\mathcal{G}1}(A) \subseteq \Phi_{m\mathcal{G}2}(A)$ *(if $\mathcal{G}_1$ and $\mathcal{G}_2$ are two grills on K)*,
(3) *If $A \notin \mathcal{G}$ then $\Phi_m(A) = \phi$.*

Proposition 2.13. [17] *Let (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space and a grill $\mathcal{G}$ on K. Then for all $A, B \subseteq K$.*

(1) $\Phi_m(A \cup B) = \Phi_m(A) \cup \Phi_m(B)$,
(2) $\Phi_m(\Phi_m(A)) \subseteq \Phi_m(A) = m\text{-}cl(\Phi_m(A)) \subseteq m\text{-}cl(A)$.

Theorem 2.14. [17] *Let $\mathcal{G}$ be a grill on a micro topological spaces (K, $\mathcal{N}$, $\mathcal{M}$).*

(1) *If* $U \in \mathcal{M}$ *then* $U \cap \Phi_m(A) = U \cap \Phi_m(U \cap A)$, *for any* $A \subseteq K$.

(2) *If* $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$, *then for all* $U \in \mathcal{M}$, $U \subseteq \Phi_m(U)$.

(3) $\Phi_m(A) \setminus \Phi_m(B) = \Phi_m(A \setminus B) \setminus \Phi_m(B)$, *for any* $A, B \subseteq K$

Corollary 2.15. [17] *Let* $\mathcal{G}$ *be a grill on a space* $(K, \mathcal{N}, \mathcal{M})$ *and suppose* $A, B \subseteq K$, *with* $B \notin \mathcal{G}$. *Then* $\Phi_m(A \cup B) = \Phi_m(A) = \Phi_m(A \setminus B)$.

Definition 2.16. [17] *Let* $\mathcal{G}$ *be a grill on a space* $(K, \mathcal{N}, \mathcal{M})$. *We define a map* $\Psi_m : \wp(K) \to \wp(K)$ *by* $\Psi_m(A) = A \cup \Phi_m(A)$, *for all* $A \in \wp(K)$.

Theorem 2.17. [17] *The above map* Ψ_m *satisfies the following conditions:*

(1) $A \subseteq \Psi_m(A)$, $\forall\ A \subseteq K$,

(2) $\Psi_m(\phi) = \phi$ *and* $\Psi_m(K) = K$,

(3) *If* $A \subseteq B$ $(\subseteq K)$, *then* $\Psi_m(A) \subseteq \Psi_m(B)$,

(4) $\Psi_m(A \cup B) = \Psi_m(A) \cup \Psi_m(B)$,

(5) $\Psi_m(\Psi_m(A)) = \Psi_m(A)$.

Now $\mathcal{M}^*$ $(\mathcal{G}, \mathcal{M}) = \{\ U \subset K\ ; \Psi_m\ (K \setminus U) = K \setminus U\}$, *where for any* $A \subseteq K$, $\Psi_m(A) = A \cup \Phi_m(A) = m_{\mathcal{G}}$*-cl(A).* $\mathcal{M}^*$ $(\mathcal{G}, \mathcal{M})$ *is called* $m_{\mathcal{G}}$*-topology which is finer than* $\mathcal{M}$ *(we simply write* $\mathcal{M}^*$ *for* $\mathcal{M}^*$ $(\mathcal{G}, \mathcal{M})$. *The elements of* $\mathcal{M}^*$ $(\mathcal{G}, \mathcal{M})$ *are called* $m_{\mathcal{G}}$*-open set and the complement of an* $m_{\mathcal{G}}$*-open set is called is called* $m_{\mathcal{G}}$*-closed set. Here* $m_{\mathcal{G}}$*-cl(A) and* $m_{\mathcal{G}}$*-int(A) will denote the closure and interior of A in* $(K, \mathcal{M}^*)$.

Definition 2.18. [17] *A basis* $\beta(\mathcal{G}, \mathcal{M})$ *for* $\mathcal{M}^*$ *can be described as follows:* $\beta(\mathcal{G}, \mathcal{M}) = \{V \setminus F : V \in \mathcal{M}, F \notin \mathcal{G}\}$.

Theorem 2.19. [17] *Let* $(K, \mathcal{N}, \mathcal{M})$ *be a micro topological space and* $\mathcal{G}$ *be a grill on* K. *Then* $\beta(\mathcal{G}, \mathcal{M})$ *is a basis for* $\mathcal{M}^*$.

Corollary 2.20. [17] *For any grill $\mathcal{G}$ on a micro topological space (K, $\mathcal{N}$, $\mathcal{M}$), $\mathcal{M} \subseteq \beta(\mathcal{G}, \mathcal{M}) \subseteq m_{\mathcal{G}}$.*

Lemma 2.21. [17] *(1) If $\mathcal{G}_1$ and $\mathcal{G}_2$ are two grills on a space K with $\mathcal{G}_1 \subseteq \mathcal{G}_2$, then $m_{\mathcal{G}2} \subseteq m_{\mathcal{G}1}$,*

(2) If $\mathcal{G}$ is a grill on a space K and $B \notin \mathcal{G}$, then B is $m_{\mathcal{G}}$-closed in (K, $\mathcal{M}^*$),

(3) For any subset A of a space K and any grill $\mathcal{G}$ on K, $\Phi_m(A)$ is $m_{\mathcal{G}}$-closed.

Definition 2.22. [17] *A subset A of a micro grill topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$) is $m_{\mathcal{G}}$ dense in itself (resp. $m_{\mathcal{G}}$-perfect, $m_{\mathcal{G}}$-closed) if $\Psi_m(A) = A$ or equivalently if $A \subseteq \Phi_m(A)$ (resp. $A = \Phi_m(A)$, $\Phi_m(A) \subseteq A$).*

3. Micro grill forms of open sets

Definition 3.1. *Let A subset A of an micro grill topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$) is said to be,*

(1) *Φ_m-open if $A \subset$ m-int($\Phi_m(A)$),*

(2) *$\mathcal{G}$-m-α-open if $A \subseteq$ m-int(Ψ_m(m-int(A))),*

(3) *$\mathcal{G}$-m-pre-open if $A \subseteq$ m-int($\Psi_m(A)$),*

(4) *$\mathcal{G}$-m-semi-open if $A \subseteq \Psi_m$(m-int(A)),*

(5) *$\mathcal{G}$-m-b-open if $A \subseteq$ m-int($\Psi_m(A)$) $\cup\ \Psi_m$(m-int(A)),*

(6) *$\mathcal{G}$-m-β-open if $A \subseteq \Psi_m$(m-int($\Psi_m(A)$)),*

(7) *$\mathcal{G}$-m-regular-open if m-int($\Psi_m(A)$) $= A$,*

(8) *$\mathcal{G}$-m-π-open if A is the finite union of $\mathcal{G}$-m-regular-open sets.*

The complement of above mentioned micro grill open sets are called their respective micro grill closed sets.

Theorem 3.2. *For a subset of an micro grill topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$) the following hold:*

(1) *Every $\mathcal{G}$-m-α-open set is Micro α-open.*

(2) *Every $\mathcal{G}$-m-semi-open set is Micro semi-open.*

(3) *Every $\mathcal{G}$-m-β-open set is Micro β-open.*

(4) *Every $\mathcal{G}$-m-pre-open set is Micro pre-open.*

(5) *Every $\mathcal{G}$-m-b-open set is Micro b-open.*

Proof. (1) Let A be an $\mathcal{G}$-m-α-open set. Then we have A $\subseteq$ m-int(Ψ_m(m-int(A))) = m-int(Φ_m(m-int(A)) $\cup$ m-int(A)) $\subseteq$ m-int(Φ_m(m-int(A)) $\cup$ m-int(m-int(A)) $\subseteq$ m-int(Φ_m(m-int(A)) $\cup$ (m-int(A)) $\subseteq$ m-int(Φ_m(m-int(A))) $\subseteq$ m-int(m-cl(m-int(A))) = Mic-int(Mic-cl(Mic-int(A))). Hence A is Micro α-open.

(2) Let A be an $\mathcal{G}$-m-semi-open set. Then we have A $\subseteq$ Ψ_m(m-int(A)) $\subseteq$ Φ_m((m-int(A))) $\cup$ (m-int(A)) $\subseteq$ m-cl(m-int(A)) $\cup$ m-cl(m-int(A)) $\subseteq$ m-cl(m-int(A)) = Mic-cl(Mic-int(A)). Hence A is Micro semi-open.

(3) Let A be an $\mathcal{G}$-m-β-open set. Then we have A $\subseteq$ Ψ_m(m-int(Ψ_m(A))) $\subseteq$ Ψ_m(m-int(A $\cup$ Φ_m(A)) $\subseteq$ Ψ_m(m-int(A $\cup$ m-cl(A))) $\subseteq$ Ψ_m(m-int(m-cl(A) $\cup$ m-cl(A)))$\subseteq$ Ψ_m(m-int(m-cl(A))) $\subseteq$ (Φ_m(m-int(m-cl(A))) $\cup$ (m-int(m-cl(A))) $\subseteq$ m-cl(m-int(m-cl(A))) = Mic-cl(Mic-int(Mic-cl(A))). Hence A is Micro β-open.

(4) Let A be a $m_{\mathcal{G}}$G-pre-open. Then A $\subset$ m-int(Ψ_m(A)) = m-int(A $\cup$ Φ_m(A)) $\subset$ m-int(A $\cup$ m-cl(A)) = m-int(m-cl(A)). Therefore, A is a Micro pre-open set.

(5) Let A be an $\mathcal{G}$-m-b-open set. Then we have A $\subseteq$ m-int(Ψ_m(A)) $\cup$ Ψ_m(m-int(A)) $\subseteq$ m-int(Φ_m(A) $\cup$ A) $\cup$ (Φ_m(m-int(A)) $\cup$ m-int(A)) $\subseteq$ m-int(m-cl(A) $\cup$ A) $\cup$ Φ_m(m-int(A)) $\cup$ m-int(A) $\subseteq$ m-int(m-cl(A)) $\cup$ (m-cl(m-int(A)) $\cup$ m-cl(m-int(A)) $\subseteq$ m-int(m-cl(A)) $\cup$ (m-cl(m-int(A)) $\subseteq$ m-int(m-cl(A)) $\cup$ m-cl(m-int(A)). Hence A is Micro b-open. □

Theorem 3.3. *For a subset of an micro grill topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$) the following hold:*

(1) *Every $\mathcal{G}$-m-α-open set is $\mathcal{G}$-m-pre-open.*

(2) *Every $\mathcal{G}$-m-α-open set is $\mathcal{G}$-m-semi-open.*

(3) *Every $\mathcal{G}$-m-pre-open set is $\mathcal{G}$-m-b-open.*

(4) *Every $\mathcal{G}$-m-pre-open set is $\mathcal{G}$-m-β-open.*

(5) *Every $\mathcal{G}$-m-b-open set is $\mathcal{G}$-m-β-open.*

(6) *Every $\mathcal{G}$-m-semi-open set is $\mathcal{G}$-m-b-open.*

Proof. The proof is straightforward from the definitions. □

Theorem 3.4. *Every Φ_m-open set A is $\mathcal{G}$-m-pre-open.*

Proof. Let A be a Φ_m-open. Then A $\subset$ m-int(Φ_m(A)) $\subset$ m-int(A $\cup$ Φ_m(A)) = m-int(Ψ_m(A)). Therefore A is $\mathcal{G}$-m-pre-open. □

Proposition 3.5. *Every micro-open set of a micro grill topological space is $\mathcal{G}$-m-α-open.*

Proof. Let A be any micro-open set. Then we have A = m-int(A). But A $\subseteq$ Ψ_m(A) $\subseteq$ m-int(Ψ_m(A)) $\subseteq$ m-int(Ψ_m(m-int(A))). Hence A is $\mathcal{G}$-m-α-open. □

Example 3.6. *Let $K = \{a, b, c\}$ with $K / R = \{\{a, b, c\}\}$ and $X = \{b, c\}$. The nano topology $\mathcal{N} = \{\phi, K\}$. If $\mu = \{a\}$ then the micro topology $\mathcal{M} = \{\phi, \{a\}, K\}$ and $\mathcal{G} = \{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, K\}$. Then $\mathcal{G}$-m-α-open sets are ϕ, K, $\{a\}$, $\{a, b\}$, $\{a, c\}$. It is clear that $\{a, b\}$ is $\mathcal{G}$-m-α-open but not micro-open.*

Remark 3.7. *micro open set and Φ_m-open set are independent of each other.*

Example 3.8. *Let $K = \{a, b, c\}$ with $K / R = \{\{b\}, \{a, c\}\}$ and $X = \{b\}$. The nano topology $\mathcal{N} = \{\phi, \{b\}, K\}$. If $\mu = \{b, c\}$ then the micro topology $\mathcal{M} = \{\phi, \{b\}, \{b, c\}, K\}$ and $\mathcal{G} = \{\{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, K\}$. Then Φ_m-open sets are ϕ, K. It is clear that $\{b\}$ is micro open but it is not Φ_m-open set.*

Example 3.9. *Let* $(K, \mathcal{N}, \mu, \mathcal{M}, \mathcal{G})$ *be defined as an Example 3.6. Then* Φ_m*-open sets are* ϕ, K, $\{a\}$, $\{a, b\}$, $\{a, c\}$. *It is clear that* $\{a, c\}$ *is* Φ_m*-open set. but it is not micro-open.*

Remark 3.10. *From the above discussions and known results in [[9], [30]] we obtain the following diagram where* $A \rightarrow B$ *represents* A *implies* B*, but not conversely.*

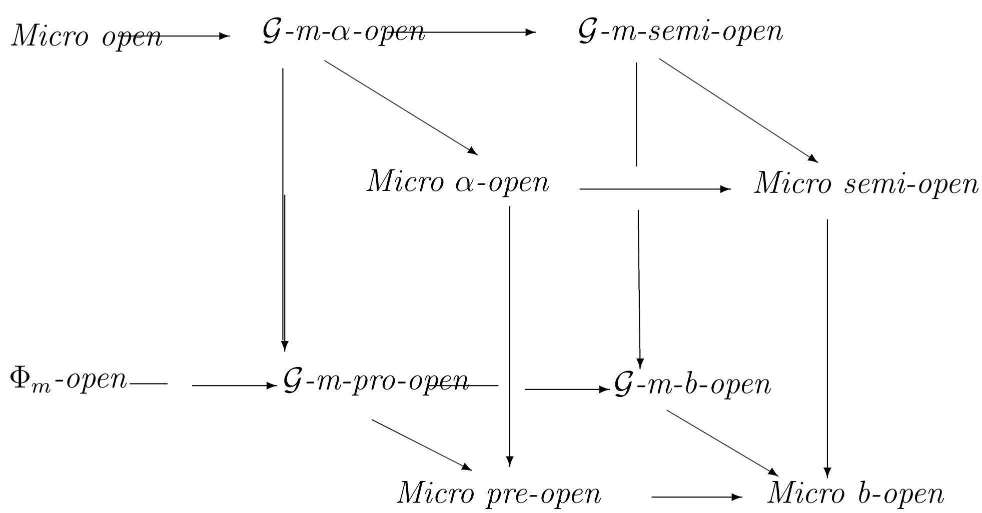

Diagram-I

$\mathcal{G}$-m-b-open $\rightarrow$ Micro b-open

$\downarrow$ $\downarrow$

$\mathcal{G}$-m-β-open $\rightarrow$ Micro β-open

Diagram-II

None of the above implications is reversible as shown in the remaining examples and in the related paper [[9], [30]].

Remark 3.11. *For a subset of an micro grill topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$), we have following implications:*

$$\mathcal{G}\text{-}m\text{-}regular\text{-}open \Rightarrow \mathcal{G}\text{-}m\text{-}\pi\text{-}open \Rightarrow Micro\ open$$

Diagram-III

Theorem 3.12. *For a subset A of an micro grill topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$), the following properties hold.*

(1) *Every $\mathcal{G}$-m-regular-closed set is $\mathcal{G}$-m-α^*-open and $\mathcal{G}$-m-semi-open.*

(2) *Every $\mathcal{G}$-m-regular-closed set is $m_{\mathcal{G}}$-perfect.*

Proof. (1) Let A be $\mathcal{G}$-m-regular-closed set. Then, we have Ψ_m(m-int(A)) = m-int(A) $\cup$ Φ_m(m-int(A)) = m-int(A) $\cup$ m-int(A) = A. Thus, m-int(Ψ_m(m-int(A))) = m-int(A) and A $\subset$ Ψ_m(m-int(A)). Therefore, A is $\mathcal{G}$-m-α^*-open and $\mathcal{G}$-m-semi-open.

(2) Let A be $\mathcal{G}$-m-regular-closed set. Then we have A = Ψ_m(m-int(A)). Since m-int(A) $\subset$ A, Ψ_m(m-int(A)) $\subset$ Ψ_m(A) [by Thoerem 2.17 (5)]. Then we have A = Ψ_m(m-int(A)) $\subset$ Ψ_m(A). On the other hand by Theorem 2.17 it follows from A = Ψ_m(m-int(A)) that Ψ_m(A) = Ψ_m(Ψ_m(m-int(A))) $\subset$ Ψ_m(m-int(A)) = A. Therefore, we obtain A = Ψ_m(A). Hence, A is $m_{\mathcal{G}}$-perfect. □

Example 3.13. *Let K = $\{a, b, c\}$ with K / R= $\{\{a, b, c\}\}$ and X= $\{b, c\}$. The nano topology $\mathcal{N}$ = $\{\phi, K\}$. If μ = $\{a, b\}$ then the micro topology $\mathcal{M}$ = $\{\phi, \{a, b\}, K\}$ and $\mathcal{G}$ = $\{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, K\}$. Then $\mathcal{G}$-m-regular-closed set are ϕ, K ; $\mathcal{G}$-m-α^*-open sets are ϕ, K, $\{a\}$, $\{b\}$, $\{a, c\}$, $\{b, c\}$: $\mathcal{G}$-m-semi-open sets are ϕ, K, $\{a, b\}$. Then (i) A = $\{c\}$, it is clear that A is an $\mathcal{G}$-m-α^*-open set which is not $\mathcal{G}$-m-regular-closed. For A = $\{c\}$ $\subset$ K, since m-int(A) = ϕ, Ψ_m(m-int(A)) = ϕ and* *hence* Ψ_m*(m-int(A))* = *m-int(A)* ∪ Φ_m*(m-int(A))* = ϕ. *Thus, we have m-int(*Ψ_m*(m-int(A)))* = ϕ = *m-int(A) and hence A is an* $\mathcal{G}$*-m-*α^**-open set. On the other hand, since* Ψ_m*(m-int(A))* = $\phi \neq \{c\}$ = *A, A is not* $\mathcal{G}$*-m-regular-closed.*

(2) A = $\{a,\ b\}$. *Then A is* $\mathcal{G}$*-m-semi-open set which is not* $\mathcal{G}$*-m-regular-closed. For A* = $\{a,\ b\} \subset K$, *since m-int(A)* = $\{a,\ b\}$, Ψ_m*(m-int(A))* = *K and hence* Ψ_m*(m-int(A))* = *m-int(A)* ∪ Φ_m*(m-int(A))* = $K \supset \{a,\ b\}$ = *A. Hence, A is a* $\mathcal{G}$*-m-semi-open set. On the other hand,* Ψ_m*(m-int(A))* = $K \neq \{a,\ b\}$ = *A and hence A is not* $\mathcal{G}$*-m-regular-closed.* □

Example 3.14. *Let (K,* $\mathcal{N}$*,* μ*,* $\mathcal{M}$*,* $\mathcal{G}$*) be defined as Example 3.6. Then* $m_{\mathcal{G}}$*-perfect are* ϕ*, K,* $\{b,\ c\}$ *;* $\mathcal{G}$*-m-regular-closed sets are* ϕ*, K. Here, A* = $\{b,\ c\}$*, it is clear that A is* $m_{\mathcal{G}}$*-perfect but not* $\mathcal{G}$*-m-regular-closed. For A* = $\{b,\ c\} \subset K$, $\Psi_m(A) = \{b,\ c\}$ = *A and hence A is* $m_{\mathcal{G}}$*-perfect. On the other hand, since m-int(A)* = ϕ *and* $\phi \notin \mathcal{G}$ *we have* Ψ_m*(m-int(A))* = $\Psi_m(\phi) = \phi \neq \{b,\ c\}$ = *A. Hence A is not* $\mathcal{G}$*-m-regular-closed.*

Corollary 3.15. *Every* $\mathcal{G}$*-m-regular-closed set is* $m_{\mathcal{G}}$*-closed and* $m_{\mathcal{G}}$ *dense in itself.*

Proof. The proof is obvious from Theorem 3.12. □

Remark 3.16. *From the above discussions and known results in [17] we obtain the following diagram where A → B represents A implies B, but not conversely.*

$m_{\mathcal{G}}$*-closed* $\mathcal{G}$*-m-*α^**-open* $\mathcal{G}$*-m-*α*-open*

↑ ↑ ↓

$m_{\mathcal{G}}$*-perfect* ⟵ $\mathcal{G}$*-m-regular-closed* ⟶ $\mathcal{G}$*-m-semi-open*

↓

$m_{\mathcal{G}}$ *dense in itself*

Diagram-IV

4. Conclusion

We introduced and studied Φ_m-open, $\mathcal{G}$-m-α-open, $\mathcal{G}$-m-pre-open, $\mathcal{G}$-m-semi-open, $\mathcal{G}$-m-b-open, $\mathcal{G}$-m-β-open, $\mathcal{G}$-m-regular-open, $\mathcal{G}$-m-π-open, which are simple forms of micro open sets in an micro grill topological spaces. We characterize the relations between them and the related properties. The results of this study may be help in many reserches.

Acknowledgement

It is our pleasant duty to thank referees for their useful suggestions which helped us to improve our manuscript.

Acknowledgement

The authors would like to thank the editors and the anonymous reviewers for their valuable comments and suggestions which have helped immensely in improving the quality of manuscript.

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