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A Search On Non-distinct Integer Solutions To Cubic Diophantine Equation with Four Unknowns

$$x^2 - xy + y^2 + 4w^2 = 8z^3$$

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Abstract: The non-homogeneous cubic diophantine equation with four unknowns given by $x^2 - xy + y^2 + 4w^2 = 8z^3$ is analyzed for its non-zero non-distinct integer solutions through applying the linear transformtions .

Keywoeds: Cubic equation with four unknowns,Non-Homogeneous cubic,N0n-distinct integral solutions

Introduction:

The cubic diophantine equations are rich in variety and offer an unlimited field for research [1,2]. In particular, refer [3-24] for a few problems on cubic equation with 3 and 4 unknowns for obtaining non-zero distinct integer solutions.It seems that much work has not been done towards the determination of non-zero non-distinct integer solutions.Towards this end,this paper concerns with non-homogeneous cubic diophantine equation with four unknowns given by $x^2 - xy + y^2 + 4w^2 = 8z^3$ for determining its infinitely many non-zero non- distinct integral solutions by employing the linear transformations.

Method of Analysis:

The non-homogeneous cubic equation with four unknowns under consideration is

$$x^2 - xy + y^2 + 4w^2 = 8z^3 \tag{1}$$

The above equation is studied for finding its non-zero non- distinct integer solutions through different ways as presented below:

Way :1

The choice

$$x = y = w \tag{2}$$

in (1) leads to

$$5w^2 = 8z^3$$

which is satisfied by

$$z = 10\alpha^2 \ , w = \pm 40\alpha^3$$

Thus,the quadruple

$$(x, y, z, w) = (\pm 40\alpha^3, \pm 40\alpha^3, 10\alpha^2, \pm 40\alpha^3)$$

gives the non-zero non-distinct integer solutions to (1).

Way :2

Consider the choice

$$x = y, z = w \tag{3}$$

Using (3) in (1),the resulting equation is

$$y^2 = 4w^2(2w - 1)$$

which is satisfied by

$$w = 2k^2 - 2k + 1,$$
$$y = \pm 2(2k - 1)(2k^2 - 2k + 1)$$

Thus,the quadruple (x, y, z, w) representing non-zero non-distinct integer

solution to (1) is

$$(\pm 2(2k-1)(2k^2 - 2k + 1), \pm 2(2k-1)(2k^2 - 2k + 1), 2k^2 - 2k + 1, 2k^2 - 2k + 1)$$

Way :3

Introduction of the linear transformations

$$x = y = 2(u + v), \ w = u - v \tag{4}$$

in (1) leads to

$$u^2 + v^2 = z^3 \qquad (5)$$

which is satisfied by

$$u = m(m^2 + n^2)\,,\, v = n(m^2 + n^2)\,,\, z = m^2 + n^2$$

Thus,the quadruple (x, y, z, w) representing non-zero non-distinct integer

solution to (1) is

$$(2(m+n)(m^2+n^2), 2(m+n)(m^2+n^2), m^2+n^2, (m-n)(m^2+n^2))$$

Note:1

(5) is also satisfied by

$$u = a(a^2 - 3b^2)\,,\, v = b(3a^2 - b^2)\,,\, z = a^2 + b^2$$

Thus,the corresponding quadruple (x, y, z, w) representing non-zero non-distinct integer

solution to (1) is

$$(2a(a^2-3b^2)+2b(3a^2-b^2), 2a(a^2-3b^2)+2b(3a^2-b^2), a^2+b^2, a(a^2-3b^2)-b(3a^2-b^2))$$

Way :4

Substituting the linear transformations

$$x = y$$

in (1),it is written as

$$y^2 + 4w^2 = 8z^3 \qquad (6)$$

which is satisfied by

$$y = m(m^2 + n^2)\,,\, 2w = n(m^2 + n^2)\,, 2z = m^2 + n^2$$

As the interest is on finding integer solutions,replacing m by $2M$ and n by $2N$, the corresponding quadruple (x, y, z, w) representing non-zero non-distinct integer

solution to (1) is

$$(8M(M^2 + N^2)\,, 8M(M^2 + N^2)\,, 2(M^2 + N^2), 4N(M^2 + N^2))$$

Note:2

Assume

$$2z = a^2 + 4b^2 \tag{7}$$

Using (7) in (6) and applying the method of factorization,define

$$y + i2w = (a + i2b)^3$$

Equating the real and imaginary parts,one obtains

$$y = a^3 - 12ab^2, w = 3a^2b - 4b^3$$

It is to be noted here that the value of z is an integer when a is even and b is either even or odd.

Thus ,when $a = 2A, b = 2B$, the corresponding quadruple (x, y, z, w) representing non-zero non-distinct integer solution to (1) is

$$(8(A^3 - 12AB^2), 8(A^3 - 12AB^2), 2A^2 + 8B^2, 8(3A^2B - 4B^3))$$

When $a = 2A, b = 2B + 1$, the corresponding quadruple (x, y, z, w) representing non-zero

non-distinct integer solution to (1) is

$$(8A^3 - 24A(2B+1)^2, 8A^3 - 24A(2B+1)^2, 2A^2 + 2(2B+1)^2, 12A^2(2B+1) - 4(2B+1)^3)$$

Conclusion

In this paper, an attempt has been made to obtain many non-zero non-distinct integral solutions to the cubic equation with four unknowns given by $x^2 - xy + y^2 + 4w^2 = 8z^3$ through employing linear transformations.The researchers may search for other choices oflinear transformations for finding non-distinct integer solutions to the considered cubic equation. As cubic equations are rich in variety, the readers may search for obtaining integer solutions to other choices of cubic equations with multivariables.

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